Invariance in Cross-Group Research

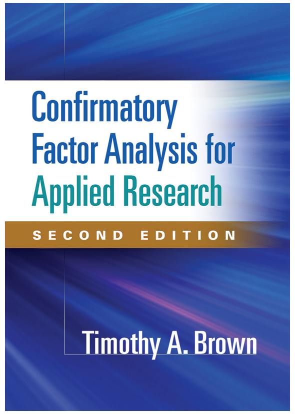
AnC MCUAAAR-V

Short Session

November 9th, 2022

Note/Disclosure: Slides structure borrows from Kate Xu's presentation titled "*Multiple group measurement invariance analysis in Lavaan*" available at this link

Why (non) invariance



- In most regression (association) work we do we are interested in group differences in means/proportions etc…
  - ***Assumption:*** is that measures are equivalent
  - ***Problem:*** Measures are potentially different across groups of interest
- Most used measures/constructs created in largely non-diverse (non-Hispanic White) populations
- Testing group equivalence needed to ensure comparability of instruments before looking at mean differences

"…*invariance evaluation is an important aspect of test development. If a test is intended to be administered in a heterogeneous population, it should be established…equivalent in subgroups of the population (e.g., gender, race). A test is said to be biased when some of its items do not measure the underlying construct comparably across groups."* Brown, T. (2015) Confirmatory Factor Analysis. 2nd Edition. pp.3

Construct operationalization

TABLE 2.3. Fundamental Steps and Procedural Recommendations for EFA

Factor extraction

- Use an estimator based on the common factor model, such as:
  - *Principal factors*: No distributional assumptions; less prone to improper solutions than maximum likelihood
  - *Maximum likelihood*: Assumes multivariate normality, but provides goodness-of-fit evaluation and, in some cases, significance tests and confidence intervals of parameter estimates

Factor selection

- Determine the appropriate number of factors by:
  - *Scree plot* of eigenvalues from the reduced correlation matrix,
  - *Parallel analysis*, and/or
  - *Goodness of model fit* (e.g., χ^2, RMSEA; see Chapter 3)

Factor rotation

- In multifactorial models, rotate the solution to obtain simple structure by:
  - Using an *oblique rotation* method (e.g., promax, geomin)

Interpret the factors and evaluate the quality of the solution

- Consider the meaningfulness and interpretability of the factors:
  - Factors should have substantive meaning and conceptual/empirical relevance
  - Rule out nonsubstantive explanations such as method effects (e.g., factors composed of reverse- and non-reverse-worded items; see Chapters 3 and 5)
- Eliminate poorly defined factors, such as:
  - Factors on which only two or three items have salient loadings
  - Factors defined by items that have small loadings (i.e., *low communalities*)
  - Factors with low *factor determinacy* (poor correspondence between the factors and their factor scores; see Grice, 2001)
- Eliminate poorly behaved items (indicators), such as:
  - Items with high loadings on more than one factor (i.e., *cross-loadings*)
  - Items with small loadings on all factors (i.e., *low communalities*)

Rerun and (ideally) replicate the factor analysis

- If items or factors are dropped in preceding step, rerun the EFA in the same sample
- Replicate the final EFA solution in an independent sample
- Consider further replications/extensions of the factor solution by:
  - Developing tentative CFA models (e.g., exploratory SEM; see Chapter 5)
  - Larger-scale CFA investigations
  - Measurement invariance evaluation in population subgroups (e.g., equivalence of solution between sexes; see Chapter 7)

Note. EFA, exploratory factor analysis; RMSEA, root mean square error of approximation; CFA, confirmatory factor analysis; SEM, structural equation modeling.

Brown, T. (2015) Confirmatory Factor Analysis. 2nd Edition. pp.34

What and When (non) invariance

“The degree to which instruments are invariant across use in different situations and with different groups” (Schmitt & Kuljanin, 2018)

“assessment [of whether an] instrument is operating in the same way and that the underlying construct has the same theoretical structure for each group.” (Dimitrov, 2010)

Applied scenarios:

- Instrument development
- Validation of existing instruments across samples (replication)
- Cross cultural work (test for different interpretation of Qs, understanding/difficulty of items, reactions, biases, etc…)
- Used in longitudinal work to test stability of constructs over time

How (non) invariance

- Through multiple group confirmatory models
  - Test equivalence of constructs across groups of interest
  - Test equality/inequality in measurement and structural parameters derived from a tested model
  - Assessment approach done through sequential fitting of a hierarchy of conditions/constraints in nested models and comparison of absolute and relative fit.

↓ in fit in nested models is evidence for (non) invariance

Fit assessment based on several measures including:

χ^2, Root Mean Square Error of Approximation (RMSEA), Comparative Fit Index (CFI), Tucker Lewis index (TLI), among others…

Which (non) invariance

- **<u>Measurement (non)invariance:</u>** Model/Factor structure, Factor means (configural), factor loadings and means (loading), factor loadings and measurement intercepts (scalar), factor loadings, measurement intercept, and residual variances (strict).

- **<u>Structural (non)invariance:</u>** Factor variances, Factor Covariances, and SEM coefficients (when SEM is tested)

Parameters Group A:

Latent factor intercepts (fixed at 0): κ1, κ2
Factor loadings: λ2, λ3, λ4, λ6, λ7, λ8
Measurement variables intercepts: τ1 – τ8
Measurement variables errors var: δ1 – δ8

Latent factor variances: ϕ11, ϕ22
Latent factor covariances: ϕ12
Regression coefficients (when SEM): β (or λ)

Equality Constraints

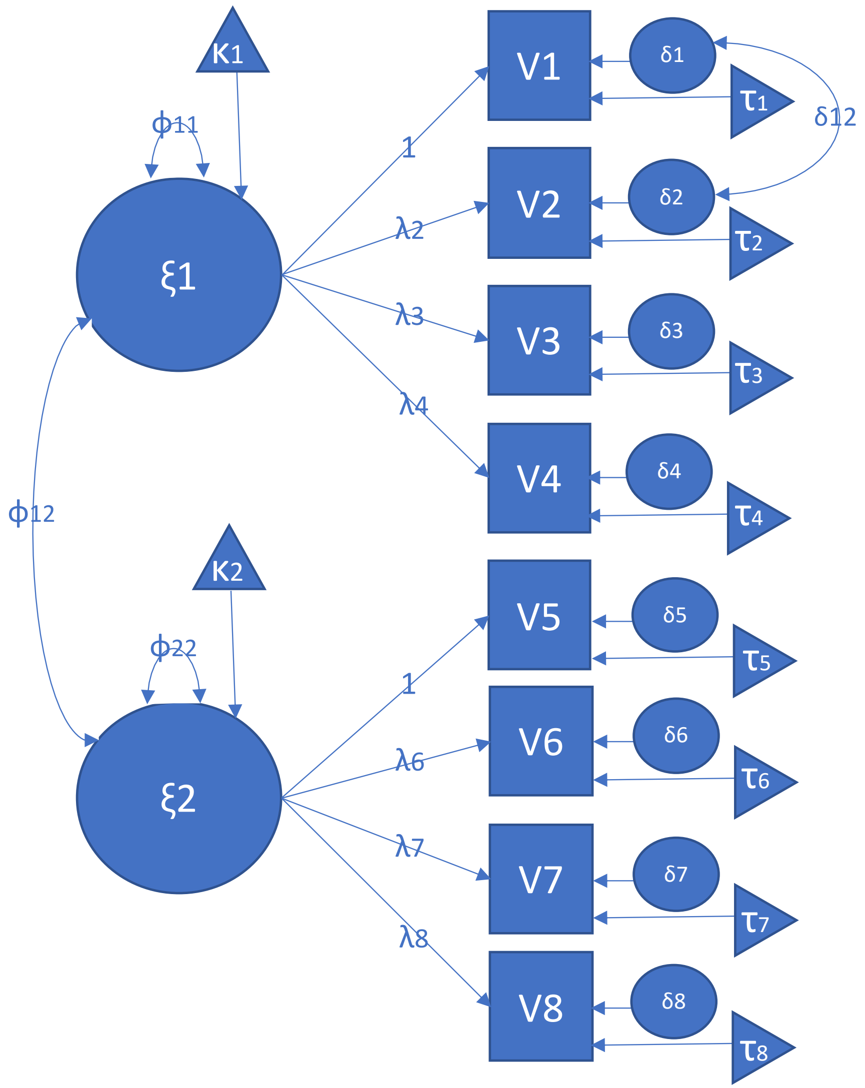
=

Parameters Group B:

Latent factor intercepts (fixed at 0): κ1, κ2
Factor loadings: λ2, λ3, λ4, λ6, λ7, λ8
Measurement variables intercepts: τ1 – τ8
Measurement variables errors var: δ1 – δ8

Latent factor variances: ϕ11, ϕ22
Latent factor covariances: ϕ12
Regression coefficients (when SEM): β (or λ)

Hypothetical CFA model – 2 constructs



ξ1 and ξ2 are cognitive constructs (memory and executive function)

Parameters:

Factor loadings (regression): λ2, λ3, λ4, λ6, λ7, λ8

Latent factor variances: ϕ11, ϕ22

Latent factor covariances: ϕ12

Latent factor intercepts (fixed at 0): κ1, κ2

Measurement variables intercepts: τ1 – τ8

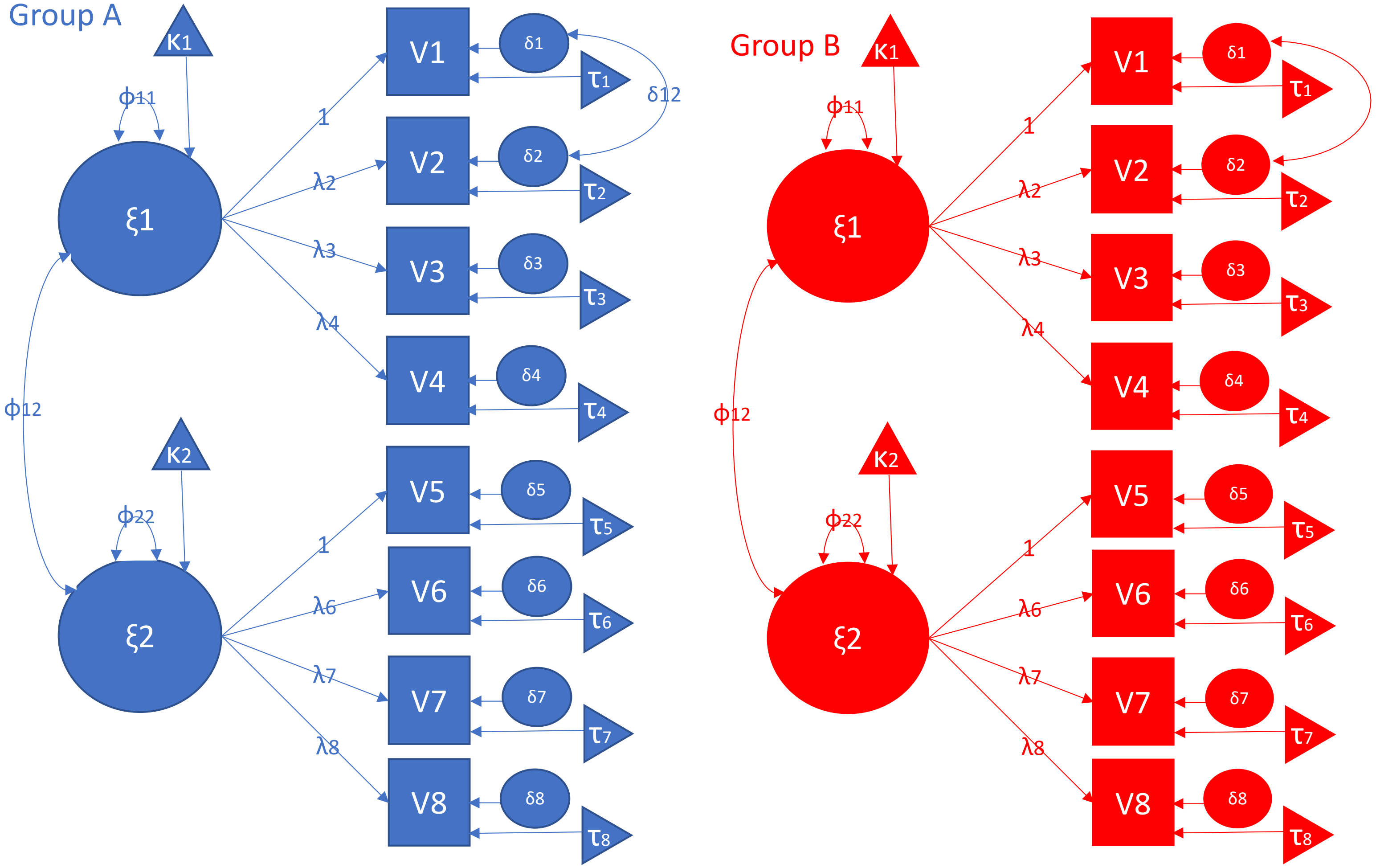
Measurement variables errors: ε1 - ε1

Name	Parameter	Matrix	Type	Description
Lambda-X	λ_X	Λ_X	Regression	Factor loadings
Theta-delta	δ	Θ_δ	Variance–covariance	Error variances and covariances
Phi	ϕ	Φ	Variance–covariance	Factor variances and covariances
Tau-X	τ_X		Mean vector	Indicator intercepts
Kappa	κ		Mean vector	Latent means
Xi (Ksi)	ξ		Vector	Names of exogenous variables

FIGURE 3.3. Latent X notation for a two-factor CFA model with one error covariance. Factor variances, factor means, and indicator intercepts are not depicted in the path diagram.

Brown, T. (2015) Confirmatory Factor Analysis. 2nd Edition. pp.48

Hypothetical CFA model – 2 constructs, 2 Groups



Parameters:
Latent factor intercepts (fixed at 0): κ1, κ2
Factor loadings: λ2, λ3, λ4, λ6, λ7, λ8
Measurement variables intercepts: τ1 – τ8
Measurement variables errors: δ1 – δ8

Latent factor variances: ϕ11, ϕ22
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Equality Constraints

Parameters:
Latent factor intercepts (fixed at 0): κ1, κ2
Factor loadings: λ2, λ3, λ4, λ6, λ7, λ8
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Measurement variables errors: δ1 – δ8

Latent factor variances: ϕ11, ϕ22
Latent factor covariances: ϕ12

Not Covered

- The mechanics of testing for invariance
- Applied approaches to testing invariance
- What to do in cases of partial invariance
- What to do when you can't show evidence for (non) invariance
- Criticisms of use invariance testing

References

(1) Xu (2012) Multiple group measurement invariance analysis in Lavaan. (Link)

(2) Dimitrov (2010) Testing for Factorial Invariance in the Context of Construct Validation. (Link)

(3) Sass, D.A., Schmitt, T.A. (2013). Testing Measurement and Structural Invariance. In: Teo, T. (eds) Handbook of Quantitative Methods for Educational Research. SensePublishers, Rotterdam. *Chapter 15 in Handbook* (Link)

(4) Brown (2015) Confirmatory Factor Analysis for Applied Research. 2nd Edition. *See especially Chapter 7* (Link)